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A Review of Fuzzy Cognitive Maps Extensions and Learning

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Abstract

Fuzzy Cognitive Maps (FCM) is a soft computing technique whose vertices and edges are fuzzy values with an inference mechanism for solving modelling problems; it has been used in modelling complex systems like industrial and process control. The concept was first introduced in 1986, with an initial learning algorithm in 1996; several works have been published on FCM methodology, learnings and applications. Fuzzy cognitive maps continue to evolve both in theory, learning algorithms and application. Many theories like intuitionistic theory, hesitancy theory, grey system theory, wavelet theory, etc., are integrated with the conventional FCM. These extensions have improved Fuzzy cognitive Maps to handle problems of uncertainty, incomplete information, hesitancy, dynamic systems and probabilistic fuzzy events. They also strengthen fuzzy cognitive Maps' modelling power for application in almost any domain. However, the compilation of the development in methodology and adaptation of FCM are either old or omitted some of the recent advances or focused on specific applications of FCM in some areas. This paper reports extension, learning and applications of FCM from the initial conventional FCM to recent extensions and some of the important features of those extensions and learning.

Keywords: Fuzzy Cognitive Map, Fuzzy Cognitive Map Learning, Fuzzy Cognitive Map extension, Advances in fuzzy cognitive maps.

1. INTRODUCTION

Fuzzy Cognitive Maps (FCM) is a machine learning algorithm with a graph structure that has vertices and edges [1]–[3]. The structure of FCM allows decomposition of a complex systems into smaller components that can be implemented as vertices in FCM models and their interaction as edges [4]. This peculiar characteristic of FCM supports its application in modelling control and engineering project. The foundational idea of FCM was a cognitive map published by the political scientist R. Axelrod in 1976 and the fuzzy logic theory which was

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earlier developed by Zadeh. Though the cognitive map was a modelling tool, Kosko [5] developed it further to include the Fuzzy element and inference mechanism. As a first attempt to produce a learning algorithm, [6] proposed Differential Hebbian Learning (DHL) which though could not lead FCM to a stable state but was an important attempt and foundation for other learning algorithms to spring forth.

Several works have been published on FCM methodology, learnings and application [7]–[10]. However, since learning algorithms is very important in modelling any domain using FCM, more attentions have been paid to review in that area with little or no compilation of the recent development in methodology and adaptation of FCM. The recent compilation of FCM methodology by [11] does not contain a good number of FCM extension. With recent reviews focusing on specific applications of FCM in some areas. As FCM continued to be applied to several problems, several researchers have both deepened and extended the initial idea to fit into certain domains and applications. This paper gives an overview of FCM with various extension and learning algorithms used in training FCM.

2. METHODOLOGY

Though several methods are available for a review of this nature, this study uses a narrative review as a form of systematic review of various FCM extensions, learning and application. A narrative pattern of review is proper for this work since various extensions, learning and applications of FCM made some improvements that may not always the quantitatively comparable [12], [13]. The method allows a qualitative identification and comparisons of various feature and improvements of FCM in the papers reviewed in other to serve as a necessary compilation of recent advances in FCM methodology.

As essential part of planning the search, a number of search keys "extension of Fuzzy Cognitive Maps, advances of Fuzzy Cognitive Maps, Fuzzy cognitive maps learning" were made in www.sciencedirect.com, scholar.google.com and www.mendeley.com. A number of search result came, however, since extensions and learnings in FCM are named after various improvement, such search yielded very few papers with such improvement. Therefore a general search of "Fuzzy cognitive maps" produced more results in several thousands of articles. As a criteria, only papers with specific improvement are reported in this work. Also, a research on conflict modelling and forecasting which began almost a decade ago and has produced the paper [14] and other two articles under various reviews was instrumental to literature on FCM methods for modelling.

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This papers here represents a more recent and elaborate compilations of FCM methods than the works of [8] which was the most recent of article the authors came across on the review of FCM.

3. RESULT AND DISCUSSION

3.1 Fuzzy Cognitive Maps

FCM vertices represent concepts, entities, factors or characteristics of the system being modelled while the edges are directed arcs that denotes the influence, connection or communication among the entities or subsystems [15]. All the values of FCM are fuzzy numbers that represent the strength of a node or edge during the inference process. A graphical illustration of FCM is given in figure 1 below. In figure1, $C_{i,}\ (i = 1 \ldots 5)$ are the values of the concepts while the $W_{ij,}(\ i, j = 1 \ldots 5)$ are the weight values of the system. For some methodologies of FCM, node values are also permitted to have discrete or binary values or state identifiers which are different from the node values [16].

3.1.1 FCM formulation

As a soft computing technique, FCM can be employed to model any system using any of the three methods: experts' knowledge or opinion [14], direct extraction of information from historical/training data [17] or hybrid which combines the historical data with experts' knowledge [1]. FCM models developed by experts' knowledge only are sometimes based on and trained with a single instance of data [18]–[22] while those developed from historical data are training with multiple instances of data [15]. The hybrid has a two-stage procedure in which experts provide the information on needed concepts and their connection matrix while the historical data is used to either validate their information and/or train the FCM model [3].

Being a fuzzy system, the experts provide system values in linguistic terms, with a further transformation into fuzzy values for inference purposes. Degrees of connections, which are in the form of linguistic variables can range from low, to moderate, and high [23]. The concepts and weight values can be of a positive or negative form as the problem domain or experts may detect [8], [10]. The inference procedure takes the form of Equation (1.1) or (1.2) [24], [25]. The equation in 1.1 is a model of self-looping or a system with memory which is common in most FCM models. The equation (1.2) does not possess memory and this is not commonly used in literature. In the equation, A_i^t is the value of node A_i at time t and w_{ji} weight values of connection between node A_j and A_i.

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$$A_i^t = f\left(A_i^{t-1} + \sum_{j \neq i, j=1}^{n} A_j^{t-1} \, w_{ji}\right) \tag{1.1}$$

$$A_i^t = f\left(\sum_{j \neq i, j=1}^{n} A_j^{t-1} \, w_{ji}\right) \tag{1.2}$$

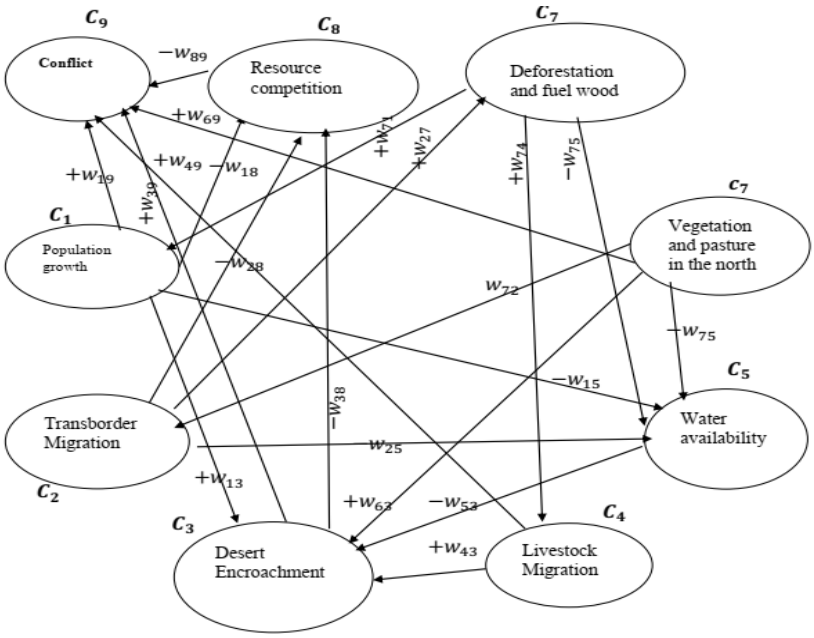

Figure 1. A simple Fuzzy Cognitive Map

3.1.2 The connection matrix and membership functions

The node and weight values are in the range of [-1, 1]. While the FCM models that use sigmoid function or bivalent function only permit node values to be between $0 \leq A_i \leq 1$; the trivalent and hyperbolic function permit node values in the range of $-1 \leq A_i \leq 1$ [7]. Generally, a connection between any two nodes A_i and A_j could be negative or positive, or no connection. A positive connection has weight values between $0 < w_{ji} \leq 1$; negative connections have weight values between $0 > w_{ji} \geq -1$, while no connection value is $w_{ji} = 0$.

The weight values are assigned by the experts based on the principle that direct or positive causation is implied if a decrease or increase in the value of A_i causes a decrease or increase in the value of A_j, while indirect or a negative causality is implied if the effect of a change in the value of A_i causes a change in the value of

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A_j in opposite directions [26]. Various membership functions ranging from trapezoidal, triangular, and Gaussian are used to fuzzify the qualitative linguistic weights from the experts. A membership function maps items in a set to members in the real interval. Various forms of aggregation or defuzzification are used to get a precise value for each edge [27], [28]. The most frequently used aggregation method for FCM model is the centre of gravity.

3.1.3 Threshold functions in fuzzy cognitive maps

Various activation functions are used for quashing the values of the output between certain ranges. Some of the function in literatures are listed in the equations (2.1) to (2.4) and include: bivalent function, trivalent function, sigmoid function and hyperbolic function [29]. Sigmoid function (equation 2.3) which quashes the values in the interval of [0, 1] is the most efficient in FCM models [30], this is due to its flexibility [31]. The bipolar function permits FCM nodes to have continues values in the interval of [-1, 1] while others may be application - specific functions.

$$f\ (x) = \begin{cases} 0, & x \leq 0 \\ 1\ , & x > 0 \end{cases} \qquad \text{(Bivalent function)} \qquad (2.1)$$

$$f\ (x) = \begin{cases} -1 & x \leq -0.5 \\ 0 & -0.5 < x < 0.5 \\ 1 & x \geq 0.5 \end{cases} \qquad \text{(Trivalent function)} \qquad (2.2)$$

$$f\ (x) = \frac{1}{1+e^{-\lambda x}} \qquad \text{(Sigmoid function)} \qquad (2.3)$$

$$f(x) = tanh(x) = \frac{e^{\lambda x} - e^{-\lambda x}}{e^{\lambda x} + e^{-\lambda x}} \qquad \text{(Hyperbolic function)} \qquad (2.4)$$

After proper specification of node and weight values, recurrent node update can follow. Repetitive node updating starts with the original state activation vector, which is then utilized to start the learning process until the FCM either reaches a fixed point or is stuck in a limit cycle or exhibits chaotic behaviour [25].

3.1.4 Extension of FCM and application

Several extensions of FCM have been made to suit certain domains; among them is Rule-based fuzzy cognitive maps by [32]. At the time rule-base was published, there was almost no strong learning algorithm for FCM; so Rule-based FCM was an extension which incorporates rules to modify the changes in the node and weight values almost together. It was a form of learning as FCM nodes are being updated. Dynamic Random Fuzzy Cognitive Maps by authors in [33] is a type of

FCM which changes its fuzzy causal web as causal patterns change and as experts update their causal knowledge. This FCM combines both inference of the node and the learning of the weight together. The node values have element of randomness which is different from regular node growth in conventional FCM. Probabilistic Fuzzy Cognitive Maps by [16] added a probability function that handles the randomness and uncertainty of fuzzy events. Every node value carries fuzzy probability while the weights are map directly to node values. Grey Fuzzy Cognitive Maps by [21] focus on modelling incomplete information problem using FCM. The integration of grey element allows the node values to be white, black or grey. This is not probability values but rather incompleteness. Table 1 below show some of FCM extensions
All the FCMs in table 1 were designed to handle some peculiar problems. FCMs has been applied in manufacturing and process control, planning, Medical Decision and Diagnosis, classification and several others. Table 2 contain a list some areas where FCM has been applied.

Table 1. FCM extensions

FCM extension	Value type	Features	source
Rule-based fuzzy cognitive maps	Crisp value	• Uses fuzzy carry accumulator with rules for causal weights calculation.	[32]
Random Fuzzy Cognitive Maps (RFCM)	Crisp value	• Nonlinear inference function and probability function for node activation. learning and inference incorporated	[34]
Dynamic Random Fuzzy Cognitive Maps	Crisp value	• Same as RFCM with dynamic weights	[35],[33]
Certainty Neuron Fuzzy Cognitive Map	Crisp number	• Several degrees of possible activation level of concept which changes based on certainty factor	[36], [37]
Genetically Evolved Certainty Neuron Fuzzy Cognitive Map	Crisp value	• Use Genetic algorithm • Combine both node inference and weight learning together.	[38]
Extended Fuzzy Cognitive Maps	Crisp value	• Conditional and time-delay weights.	[39]
Probabilistic fuzzy cognitive Maps	Crisp value	• Concept values consider as fuzzy events with state identifiers • probability Function for nodes values and • dynamic mapping of weight	[16]
Agent-Based Fuzzy Cognitive Maps (AB-FCM)	Crisp values	• multiple inference algorithms at the concept-nodes	[40]
Ontology Agent-Based Fuzzy Cognitive Maps	Same as AB-FCM	• same as AB-FCM with addition of ontology agent that manages other agents and causal relation	[41]

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FCM extension	Value type	Features	source
Fuzzy Grey Cognitive Maps(FCGM)	Interval grey number	• node values as grey numbers • Incomplete information handling	[21]
Fuzzy General Grey Cognitive Maps	General grey number	• Cope more with uncertainty than FCGM	[42]
Wavelet Fuzzy Cognitive Maps	Crisp Value	• wavelet function as transfer-function which was proved better that sigmoid	[43]
Intuitionistic Fuzzy Cognitive Maps (IFCM)	Crisp values	• Uncertainty measure and hesitancy in causal relations and concepts.	[44]
Least Square Fuzzy Cognitive Map (LSFCM)	Crisp value	• Concept values are centre of time-series data cluster found using C-means.	[45]
Genetically evolved fuzzy type-2 cognitive maps			[46]
Alpha-cut based fuzzy cognitive maps	Crisp Value	• Alpha-cut based instead general deffuzification to reduce loss of information.	[47]
Generalized fuzzy cognitive maps	Interval value	• Interval-based concept • Uses genetic algorithm to find the optimal concept values within the interval	[48]
Dynamical cognitive network(as a form of FCM)	Crisp Value	• Each concept in the DCNs can have its own value set, AS each is require	[49]
Dynamic Fuzzy Grey Cognitive Maps(DFCGM)	Real, Interval grey, and general grey numbers.	• Addition environmental variable effect on concept vales during inference. • Handle uncertainty in dynamic situation	[50]
Dynamic Fuzzy General Grey Cognitive Maps	Same as DFCGM, , multiple grey interval	• Addition environmental variable effect on concept vales during inference	[50]
Higher order IFCM	Same as IFCM	• Integration of evidential reasoning theory • operators to aggregate different differing information	[51]

3.2 FCM Learning Algorithms

FCM models are developed from data or experts' knowledge; however, as common to machine learning algorithms, FCM need to be trained else it will stabilize to undesired state [18]. The need for training FCM spur the development of various forms of algorithms that fine-tune connection matrix [18] to find the weight that leads to stable state. Three general categories can be used to group the learning algorithms for FCM: Hebbian-based, evolutionary/population-based, and hybrid [10].

Obtaining a good model from many machine learning require a large volume of training data, however, this is not necessary the case with FCM. FCM models can

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be developed and fully trained based on singles instance of data [21] or multiple instances of data [38]. When the training data is single instance of data, Hebbian learning algorithms are very suitable choice for learning FCM. The population-based algorithms mostly require multiple instances of data for good model to be achieve during learning.

Table 2. Application areas of FCM

Application Area	Source
Engineering and industrial process control	[21], [16], [52], [53], [54]
Pattern Recognitions	[55], [24],
Robotic Artificial Emotion	[56], [57]
Energy studies	[58]–[60],[52], [58]–[62]
Intelligent Security Systems	[20], [63]–[65]
Environmental pollution	[66]; [66], [67], [68]
Project management and risk analysis	[69], [70]
Livelihood analysis	[71], [72]
Military and Tactical planning	[73]
Medical Decision and Diagnosis	[1], [44]
socio-economic planning and social change	[74], [29]
Yield prediction	[75], [76]
Financial Distress Analysis	[77]
Classification	[30], [78]
Forecasting	[79]
Food security	[27]; [80]

3.2.1 Hebbian Learning Algorithms

Hebbian learning algorithms are primarily based on Heb's theory on neural firing which were modify to learn FCM. The connection matrix of these algorithms are provided by human experts in the form of linguistic variables that are transform into fuzzy values [81]. Some of the common Hebbian learnings are in table 3 with their learning equations. [6] proposed the Differential Hebbian Learning (DHL) with the learning rule in table 3. the parameter l_t represents the learning rate. The weights of outgoing edges for each concept in the connection matrix are modified only when the corresponding concept value changes. The learning technique represents one of the earliest attempt to learn FCM using Hebbian theory; however, the algorithms could not lead FCM to stable state. DHL could not get FCM to convergence since it does not include the total effect of other nodes during the learning process.

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Balanced –DHL [82] was enhancement of DHL. It modified DHL update equation of by taking into account the total effect of all concept in the network apart from the concept being updated [21]; however, it is not suitable for nonlinear problems since it was only successful for binary problem application. Below is the update equation of Balanced –DHL in table 3. Nonlinear Hebbian Learning (NHL) [83] is the most successfully and widely applied variants of Hebbian learning algorithms in learning FCM such as works of [22]. The algorithm requires experts to supply the initial weights and the appropriate connection sign for every edge. It uses the learning rule in table 3 to learn the weights that will lead FCM to stable state. An improvement of NHL by NHL [84] also produced almost same result. The variable η is the learning rate while others are as explained in the introduction. Active Hebbian Learning by[85] is a Hebbian algorithm that is highly tier to the topology of the system being model. The learning procedure requires experts to provide sequence of activation of the entire neurons in FCM while the weights is updated asynchronously in order of the node activation. The table 3 below capture the Active Hebbian Learning weight update rule. Data-Driven NHL (DDNHL), a modification of NHL by [71] utilizes the same principles as NHL while utilizing the available historical data during the learning process. DDNHL does not use the formula to determine the node values when learning; rather, it substitutes values from historical data.

Table 3. Hebbian learning rules

Algorithm	Learning rule	weakness	strength
DHL [6]	$w_{ij}(t+1) = \begin{cases} W_{ij(t)} + l_t[\Delta A_i \Delta A_j - W_{ij(t)}] & \forall A_i \neq 0 \\ W_{ij(t)} & \forall A_i = 0 \end{cases}$ $l_t = 0.1\left[1 - \frac{t}{1.1N}\right],$	• Non convergence • Learning result was inconclusive	• A pioneering search in FCM Hebbian learning
BDHL [82]	$w_{ij}(t+1) = \begin{cases} w_{ij}(t) + \frac{l_t}{N} & if\ i = j \\ w_{ij}(t) + l_t\left[\frac{\frac{\Delta A_i}{\Delta A_j}}{\sum_{k=1}^{N} \frac{\Delta A_i}{\Delta A_k}} - w_{ij}(t)\right] & if\ i \neq j\ and\ \Delta A_i \Delta A_j > (\\ w_{ij}(t) + l_t\left[\frac{-\frac{\Delta A_i}{\Delta A_j}}{\sum_{k=1}^{N} \frac{\Delta A_i}{\Delta A_k}} - w_{ij}(t)\right] & if\ i \neq j\ and\ \Delta A_i \Delta A_j < (\end{cases}$	• Applicable to binary data only • Does not use historical data to learn	• Take cognisee of total node effects, useful when dealing with binary data.
NHL [83]	$w_{ij}(k) = w_{ij}\ (k-1) + \eta c_j (C_i - C_j W_{ij}(k-1))$	• Human dependent-initial weight • Does not use historical data to learn	• Can learn with single instance of data
Improved NHL [84]	$\Delta w_{ji}^{k} = \alpha_k \Delta w_{ji}^{k-1} + \eta_k z_k^2 (1\text{-}z_k)(V_i^k - w_{ji}^{k-1} v_j^k)$ Where $z_k = {}^{1}\!/_{(1 + e^{-v_j^k})}$	Same as NHL	Same as NHL

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Algorithm	Learning rule	weakness	strength
AHL [85]	$w_{j,i}(k) = (1-\gamma)w_{ij}\ (k-1) + \eta A_j^{act}(k-1)[(A_i\ (k-1) - w_{ij}\ (k-1).(A_j^{act}(k-1))]$	• Human dependent-initial weight • Too rigid on topology of system being model • Only asynchronous systems	• Useful for asynchronous systems modelling
DD-NHL [86]	$w_{j,i}(k) = w_{ij}\ (k-1) + \eta c_j(C_i - C_j W_{ij}(k-1))$	• Human dependent-initial weight	• Uses historical data to train • Can learn with single or multiple • Better optimal result than NHL

3.2.2 Evolutionary Learning Algorithms

Biological or natural systems serve as the foundation for evolutionary algorithms. A number of such system like Ant colony, Swarm Intelligence, and genetic algorithms have all impact the computing world greatly. To learn FCM and create the appropriate weight to guide the system toward the intended state [10], they make use of the historical data with complex optimization technique. Common to all evolutionary algorithm is the fitness function that each algorithm uses to determine if a candidate solution is to be acceptable or rejected. The fitness function is mostly application specific, or as the researcher may seem necessary. Expert independence for initial weights is one benefit of evolutionary algorithms, but in comparison to Hebbian versions, they are computationally expensive and time-consuming due to employment of sophisticated optimization techniques and a significant amount of data [29].

Genetic Algorithm (GA) for learning FCM was proposed by [87]. The work solved multi-objective decision making problems using Genetic Algorithm (GA) to generate the optimal weight matrix for FCM stability. Their method uses a single state vector to generate random initial individuals with each individual representing a connection matrix. With the initial population, the algorithm continue to carry out gene mutation from one generation to another until the optimal weight matrix is found. Though it need no expert initial weight matrix, it consumes more system resource and time than Hebbian variants. The number of free parameters increases with large size of training data.

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Real-Coded Genetic Algorithm (RCGA) was used by [88] to learning FCM. The algorithm used multiple instances of data which are pair as input and system response. The large numbers of the pairs form the multiple initial maps and candidate solutions. The RCGA with certain fitness function uses random initial weights to begin search for solution. The major drawback of the method is that, it's not easily scalable as the number of parameters grow with as data grow.

FCM learning using the Ant Colony Optimization (ACO) algorithm was model by [89]. The ants in the colony communicate with one another and with their surroundings through pheromones. The ACO simulates solutions as trails in the environment, and the ants utilize pheromones to guide their search. The pheromone is used to probabilistically sample the search space and contains rules for state update and transition. Ants perform the probability creation of optimization problem solutions using a predetermined pheromone model. The process continues until the objective function's best value is found. This method requires use of substantial amount of data and involve some complexity.

As a technique for training FCMs, [90] used Extended Great Deluge Algorithm (EGDA). When determining FCM weights, EGDA employs a local-search metaheuristic approach designed to reduce subjective variables. The fact that EGDA allows subpar solutions is one of its key characteristics. [91] Proposed a multi-objective evolutionary algorithm (MOEA-FCM) to learn several FCM models. This was a good attempt to learn FCM with lower density and also making the solution to fit well - varying the densities at the same time reducing data error. Several candidate solutions are formed from the training data with each having different densities. Then an evolutionary approach is used to find solution. However, the lower the density the higher the possibilities of data error. This solution method fits well into problems of having lower connection weight number which is explainable solution.

Particle Swarm Optimization (PSO) was proposed by [92] to learn FCM They applied swarm intelligence algorithm as a computational technique for computing weight that will drive the Fuzzy Cognitive Map to desired steady states. The concepts of FCM model are model to be swarms that have position, velocity, direction and other parameter and interacting a universe. Though an evolutionary algorithm, it relies on the initial expert's weight and specification of learning boundary for the weight. [23] proposed Evolutionary Multitasking Fuzzy Cognitive Map Learning with multi-objective of error function and sparsity of FCMs. It learns several FCMs together and perform gene transfer base on similar patterns found. Experiments on varying number of nodes, densities and activation functions showed that the algorithm can learn large-scale FCMs with low errors. However, the rigorous mathematical computation associated with evolutionary algorithm and the need for various free parameters to guess makes the algorithm complex for application. [93] develop a framework based on the least absolute

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shrinkage and selection operator (lasso) termed LASSO-FCM to learn FCMs. The algorithm formalized learning produce as a sparse signal reconstruction problems and decompose the task of learning FCMs into learning local connections of nodes individually. LASSOFCM is efficient for various sizes and densities of FCM. The focus of the work was more on varying the size and density rather than more optimal solution, however, only a single map is learned. [94] Proposed Interactive Evolutionary Optimization of Fuzzy Cognitive Maps termed IEO-FCM method. This is an expert-driven optimization FCM optimization. It a modification of Interactive Evolutionary Optimization to learn FCM. As a population-based learning, the algorithm uses a random generation of weights for the initial population, however, it limits the initial maps to 15 and needs expert's evaluation for transition from one generation to another instead of a fitness function. The need for human intervention weakens the efficiency of the algorithm. [95] Proposed a niching-based multi-modal multi-agent genetic algorithm learning FCM (NMMMAGA-FCM). It combined multi-agent genetic algorithm and niching methods learn multiple FCMs concurrently. Just like LESSO-FCM, The algorithm rely on leaning specific column of each FCM through random initial connection matrix. The optimal solution is chosen based on the best learned result of each column.

Another learning algorithm called Modified Asexual Reproduction Optimization (MARO) for was proposed by [96] MORO is closely a genetic algorithm-variants that accepts the bud even if it is a little worse than the parent. This means that the new generated solution is accepted and search continues even if it is not better than previous solution. The algorithm is judge to be fast, though it produces only single solution; unlike population-based algorithms which produced multiple solution and choose the optimal. [97] employed a Rapid and Robust Learning Method with Maximum Entropy for learning FCM with noisy data. The work attempted learning of large FCM with sparse weight or density. The approach directly invoke existing convex optimization methods to find the weight values. However, direct application of complex optimization techniques in learning is often criticize due the complexity. [98] used Pseudoinverse Learning of Fuzzy Cognitive Maps. It uses Moore-Penrose inverse method to directly compute the weight parameter from historical data and node values. It's free of guessing parameters associated with many algorithms, however, the algorithm does not really perform any form of usual learning but rather a direct estimation of the weight values.

A divide and conquer algorithm is also used to learn large number of node in FCM by [99]. The learning procedure has two features: data-divider and FCMs fusion which average the weights from all the final result. It utilizes real coded genetic algorithm (RCGA) to learn and merge multiple FCM models that are computed from subsets of the original data. The quality of the result is better that Hebbian based learnings but slower in learning

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3.2.3 Hybrid Learning algorithms

Hybrid algorithms combine the two approaches of Hebbian and evolutionary strategies to learn the connection weighs. Experts' weights are needed as the initial connection matrix while combine Hebbian and evolutionary are used to learn the FCM. Examples of hybrid learning is Ensemble based learning approaches was used by [53], with bagging and boosting to learn FCM. DD-NHL was used as a base classifier. However, the work still relied on expert input for initial weight, with every candidate classifier using the same weight for training, though historical data was used for training the model. [76] Proposed a hybrid learning FCM that combines Data Driven Nonlinear Hebbian Learning (DDNHL) algorithm and Genetic Algorithm (GA) called FCM-DDNHL-GA. The initial connection matrix from experts is used to trained FCM using DD-NHL, the weight from DD-NHL-FCM is used as initial population for the GA. The advantage of this is the fact that the initial weight matrix from DD-NHL as the initial population is closer to weights that can easily reach stable state faster than regular genetic algorithm.

Deterministic Learning of Hybrid FCM by [31] Nápoles, Jastrz and Mosquera, (2020) combined historical data with expert weights to learn FCM. The weight learning makes use of Fuzzy Cognitive Map architecture in which experts are requested to define the interaction among the input neurons and Moore-Penrose inverse compute the weights among input and output neurons. However, the learning is highly tied to a specific topology of FCM, making it unfit for application in many other type of problems. The historical data is also based on system simulation while learning, this could be error prone and weak.

3.3 Training data

Every intelligent algorithm depends on data for training to model a system, however, some are heavily dependent on data for good models to be derive. Algorithms like Neural networks and deep learning require millions of training data but FCM is different. FCM model can be developed and trained using a single instance of data -this is the case with many Hebbian-based algorithms [21], [83], [85], [100], [101]. The populations-based algorithms require large data to train, however, literature from FCM research show that with few instances of data, FCM can be train with good model derived [100], [102]. The advantage of FCM adoption to solve soft computing problem is in modelling system efficiently using very limited data.

3.4 Fitness function algorithms

While it is common for many other machine learning algorithms to measure accuracy level using F1 score, positive and negative prediction, sensitivity, etc; the

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accuracy of FCM model can go beyond these. FCM models are measure on how best the learned model fits the data or system being mode.

Fitness function evaluates the error of a given FCM by the computation of cumulative prediction error, i.e., the sum of differences between the data generated by the candidate FCM and the original data or map. The choice of fitness functions for a particular training algorithm is influenced by the domain/problem or system being modelled. The fitness function for each chromosome (candidate solution or map) is calculated by simulating the each candidate solution from the initial state vector and simulating original map. The responses of the both the candidate solution and the original map is then compared to find how the candidate solution fits or model the data. There are several fitness functions used to evaluate FCM model performance in different datasets, some are given below.

i. **In-sample error**. In this evaluation, the data used in training is used to test the resultant FCM model and weights in other to measure how it fits into the training data [96]. It is very important because it measures the ability of the resultant model to remember or recognize previously seen data. The error is measure as the difference between the input data and data produced by simulated FCM model from the initial state vector. The formula is given in equation (3).
ii. ***Out-of-sample error.*** This measures the generalisation capabilities of the candidate FCM. To compute this criterion, the following formula is used.
iii. **Error-l** which is the average of the squared difference between the two-time series and is defined as follows.

$$\text{in sample error} = \frac{1}{(k\text{-}1)N} \sum_{t=1}^{K} \sum_{n=1}^{N} \left| A_n(t) - \hat{A}_n(t) \right| \quad (3)$$

$$\text{out of sample error} = \frac{1}{p(k\text{-}1)N} \sum_{p=1}^{P} \sum_{t=1}^{k-1} \sum_{n=1}^{N} \left| A_n^p(t) - \hat{A}_n^p(t) \right| \quad (4)$$

$$\text{Erro-1} = \frac{1}{R.(k\text{-}1)N} \sum_{t=1}^{K} \sum_{n=1}^{N} \left(A_n(t) - \hat{A}_n(t) \right)^2 \quad (5)$$

In the equation (3) to (5), $A_n(t)$ is node *n values* at iteration *t* in the input data, $\hat{A}_n(t)$ the value of node *n* at iteration t from simulation of the candidate FCM, *K* the number of input data points and *N* the number of nodes. P is the total number of different initial state vectors, and $A_n^p(t)$ is the value of node n at iteration t for data produced by input FCM started from pth initial state vector. Similarly, $\hat{A}_n^p(t)$ is the value of a node n at iteration t for data produced by candidate FCM started from pth initial state vector, R is the number of starting state vectors chosen at random.

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Stach *et al.* (2005) used $F = \frac{1}{(a(error)+1}$, with a =10.00 and error = equation (5) as fitness function. $f = \propto \frac{1}{\beta. \sum_{t=1}^{K}\sum_{n=1}^{N}\left(A_n(t)\text{-}\hat{A}_n(t)\right)^2}$ was used as a fitness function by Stach, Kurgan and Pedrycz (2010) to learn FCM weight with using genetic algorithm with divide and conquer strategy, β *are* positive scaling constants. [96] used in-sample and out-of-sample error to measure how Modified Asexual Reproduction Optimization (MARO) learn FCM and several other application specific functions. A number of fitness functions could be listed, however, the choice of fitness fucnction is depedent on both the system and the goal of learning and should be designed to produce the best model or solution.

3.5 Fuzzy Cognitive Maps Simulation Tools

A number of simulations software have been developed to aid modelling of FCM, though not exhaustive, [11] give a list of some. *FCM Modeler* which supports FCM modelling process through GUI and node update procedure. This software appear to be more a generic application for conventional FCM. Another software called *FCM Designer* has feature like visual and graphical display of models. Though it can cannot be used in training FCM, however, the application have various threshold functions to choose from. *Mental Modeler* is like the previous applications but deployed as an online systems to help experts develop simple FCM but it cannot be used for training FCM since it lack learning component. *Java Fuzzy Cognitive Maps* (JFCM) as FCM modelling application allows different kind of models to be developed as reusable models. *Intelligent Expert System based on Cognitive Maps* is an advanced application which apart from modeling system using FCM, also possess learning algorithms like genetic algorithm for training FCM. The most advance of them is the *FCM Expert* which have several learning algorithms, both evolutionary and Hebbian based.

4. CONCLUSION

Fuzzy cognitive maps continue to evolve both in theory, learning algorithms and application. With many theories like intuitionistic theory, hesitancy theory, grey system theory, wavelet theory, etc., being integrated in the conventional FCM. These extensions have improved Fuzzy cognitive Maps to handle problem of uncertainty, incomplete information, hesitancy, dynamic systems and probabilistic fuzzy events. They also strengthens fuzzy cognitive Maps' modelling power for application in almost any domain. The advantage of FCM over other machine learning algorithms lies in its power to use both qualitative and quantitative data for modelling and to learn a complex system from very few instances of data. While learning algorithms have been developed as Hebbian or Population-based variants; Hebbian is simple, fast but human dependent while population-based algorithms are human-independent but consume high computational resources

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and are not easily scalable. The application areas is wide but FCM seem more efficient in system control than classifications or prediction, though some works show that with proper topology of FCM, a good application in all field is possible.

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